Cosmology with interacting dark energy

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Abstract

The early cosmic inflation, when taken along with the recent observations that the Universe is currently dominated by a low density vacuum energy, leads to at least two potential problems that modern cosmology must address. First, there is the old *cosmological constant problem*, with a new twist: the *coincidence problem*. Secondly, cosmology still lacks a model to, *a priori,* predict the observed current vacuum domination and to determine whether or not there is a future graceful exit from the consequent acceleration (as previously in the inflationary case). This constitutes (what is called here) a *dynamical problem.* In this article a framework is proposed to address these two problems, based on the premise that the background dark energy is both dynamical and interacting. The approach suggests an inflationary scenario as the initial condition to the current classical evolution of the Universe and at the same time offers a natural graceful exit to such a scenario.

I. Introduction

In the last few years, evidence has been mounting to suggest that the Universe is currently accelerating. Measurements of type Ia supernovae [1] indicate that the evolution of the Hubble parameter departs from that expected for a matter dominated Universe, and behaves as if under the influence of a negative pressure due to a smooth and dominant background dark energy. Further evidence for a vacuum-dominated Universe comes from a combination of observations: large scale structure (LSS) [2] suggests a low matter density Universe while the cosmic microwave background (CMB) anisotropy data [3] shows the density of the Universe to be virtually critical, consistent with the requirements of the inflationary scenario [4]. Successful inflation requires that the Universe start off in a vacuum-dominated state with a very large associated potential energy. The discrepancy between this initial high energy state and the currently observed low background dark energy constitutes what has been called [5] the cosmological constant problem. A new twist which further compounds this problem, and which has been named the coincidence problem, relates to the observation [1] that the current background vacuum energy density $\rho_v^0 \sim 10^{-30}\ g\,cm^{-3}$ is of the same order of magnitude as the current density ρ_m^0 of the matter fields.

Such a scenario gives rise to yet another complication which is not often mentioned. One notes that the standard model of cosmology [6], bolstered by inflation [4], has been extremely successful in explaining the evolution of the Universe in remarkable detail including, for example, the nucleosynthesis of low mass (H, He) elements [7], the origins of CMB radiation [8, 9], and LSS [2]. However, this standard model leaves some questions still open. For example, the currently observed vacuum-dominated state of the Universe could not have been specifically predicted *a priori*. Moreover, it is not known whether there will be a future graceful exit from the current acceleration (as was the case in the early Universe). Such a state of affairs reflects our ignorance in the specifics of the future dynamics of the Universe. This constitutes a dynamical problem.

Several models have been developed to address the cosmological constant problem. They include dynamical $\Lambda-$term models [10, 11], dynamical equation of state models [12], and rolling scalar field models [13]. A common feature in all these models is that the vacuum energy takes on a dynamical character, decaying from some initially large value to a small one. Such approaches have generally been aimed at dealing with the "why is Λ small now?" part of the cosmological constant problem. On the other hand, to date modern cosmology still lacks a physically justifiable solution to the coincidence problem, beyond invoking an anthropic principle [14].

In this article a potential framework for addressing both the cosmological constant problem and the dynamical problem is proposed. The framework rests on the premise that the background vacuum energy is both dynamical and interacting. This implies that the Universe can be treated as a vacuum-driven cosmic engine. Here, the vacuum energy acts as the fuel that does work on the Universe by accelerating it. As is the case with any engine, it is impossible for the Universe to convert all its fuel to mechanical work without dissipating some through irreversible processes that increase its internal energy. This is a statement of the first law of thermodynamics. In turn, the irreversible dissipations lead to a cosmic form of the second (or entropy creation, (s)) law, $s^{\mu}_{;\mu} \geq 0$. In this model, such a dissipation process is cardinal to the dynamics of the Universe, and to underscore this we propose the following "cosmic equilibrium" conjecture:

Conjecture 1 *The Universe will increase its inertia (through matter creation) as a backreaction to any influences tending to move it away from (dynamic) equilibrium.*

Facilitating this process is a (bulk viscous) creation pressure π_c, which arises as a backreaction to the spacetime acceleration. The role of this back-pressure is to build up inertia to oppose the change (acceleration) which creates it, in the first place. It does this by creating matter. Such behavior is akin to that in other equilibrium-seeking systems like, for example, electromagnetic induction as described by Lenz's law [15]; or as in the description of quark confinement (in QCD) as described in terms of asymptotic freedom [16]. The process couples the vacuum to matter through a parameter K, which we constrain using current observations, along with theoretical considerations. Thus, in this scenario, matter will be created until the cosmic acceleration is offset and the creation pressure π_c vanishes, eventually leading to matter domination. With no immediate further need for matter creation, the existing matter fields in a given comoving volume begin to dilute normally, (i.e. faster). Such redshifting of the matter fields, in time, leads to a further vacuum domination and another acceleration/creation cycle commences.

The creation process sets up oscillations in the decaying vacuum and corresponding sympathetic ones in the matter fields. By establishing bounds on their relative magnitudes, one finds that the fields track each other naturally. It is in this sense that the model offers a natural and conceptually simple explanation to both the coincidence problem and the dynamical problem. As a by-product, the approach recovers inflation as a natural initial condition to the current classical dynamics of the Universe. At the same time, it provides a rationale for a graceful exit to, not only the inflationary scenario but also, the current acceleration. Lastly, the 'cosmic equilibrium' conjecture suggests a potentially scientific ansatz to what would be considered a philosophical question, namely, "why does the universe need matter?".

This article discusses the bulk dynamics of the Universe. A complete model will require the inclusion of micro-physics to deal with such issues like what kind of matter is created during each cycle of vacuum domination. In general, though, such dissipative/creation processes should underlie the overall dynamics of the Universe. The rest of the article is organized as follows. In Section II the working equations for an interacting vacuum energy are set up. Section III discusses the evolution of the vacuum energy density in the presence of matter creating processes. Section IV lays out the framework for the evolution of the matter fields in such an environment. A potential resolution to both the coincidence problem and the dynamical problem is presented and observational tests are mentioned. Section V concludes the article. Throughout the article the terms *vacuum* energy and dark *energy* are used interchangeably to describe a background cosmic energy that is both dynamical and interacting.

II. Interacting vacuum: working equations

A. Features

In modeling an interacting vacuum, we begin by assuming a dynamical cosmological parameter, Λ with a form

$$\Lambda\left(t\right)=m_{pl}^{4}\left(\frac{l_{pl}}{a(t)}\right)^{\sigma(t)}e^{-\tau H}=\Lambda_{pl}\left(\frac{l_{pl}}{a(t)}\right)^{\sigma(t)}e^{-\tau H},2.1 \qquad (1)$$

where m_{pl} and l_{pl} are the Planck mass and length, respectively, τ is of order of the Planck time, H is the Hubble parameter and $a\left(t\right)$ is the scale factor. The power index $\sigma\left(t\right)$ is a function of time, yet to be determined. Such a form of Λ has the following features. First, it is regular and vanishes at time $t=0$, (since $H\sim\frac{1}{t}$). In this scenario, the Λ-vacuum appears to tunnel from nothingness. Thereafter, the dynamics of the early Universe $t\leq\tau$ is driven by the growing term $e^{-\tau H}$ and should be dominated by a quantum character. A rigorous discussion of the physics of this period necessitates a quantum theory of gravity. One notes that the $\Lambda\left(t\right)$ function (above) has a stationary point (max) at $\left[\ln\left(\frac{l_{pl}}{a(t)}\right)\frac{d\sigma}{dt}-\sigma\left(t\right)H-\tau\frac{dH}{dt}\right]=0$. In terms of the vacuum energy $\langle\rho_{vac}\rangle=\frac{\Lambda}{8\pi G}$, one expects that such a stationary point should occur at about $\langle\rho_{vac}\rangle\sim10^{94}\,g\,cm^{-3}$. In the immediate neighborhood of this stationary point, Λ is virtually constant and the scale factor $a\left(t\right)$ is inflationary. However, the quantity $e^{-\tau H}$ quickly approaches saturation $e^{-\tau H}\rightarrow1$ as the Hubble time H^{-1} grows. Subsequently, the dynamics of the Universe becomes increasingly classical, being driven by the $a(t)^{-\sigma(t)}$ part of Λ. One distinguishing feature of the model presented here is that the power index function $\sigma\left(t\right)$ is not a fixed constant. Further, the functional form of $\sigma\left(t\right)$ is not introduced by hand but is, instead, built during the discussion. It is this feature, in our treatment, that allows the vacuum to couple to matter.

Assuming successful inflation, then by the time the Universe is about 1 second old, the scale factor is generally expected to have grown by a factor of about 10^{28}. In our case, this means that Λ will have decayed by $10^{-28<\sigma(t)>}$, in the process, leaving behind a dense field of relativistic particles and radiation. It turns out, as will be shown later (Sec.III), that the time average value of the power index function, in the model, is given by $<\sigma\left(t\right)>=2$. As a result, the vacuum energy density does not interfere with the usual early cosmological processes like big bang nucleosynthesis (BBN) [7], but instead allows such processes to proceed as predicted by the standard model. Further, since the inflation era to date, $a(t)$ has evolved by about 10^{60} [17]. The result is that $\left[\Lambda_{now} \sim a_0^{-2}\right] \approx 10^{-120}\Lambda_{pl}$, which is consistent with observations. This provides a heuristic explanation to the "why is Λ small" part of the cosmological problem, pending the proof (later in our discussion) that $<\sigma(t)>= 2$.

In the remaining part of this article, tools are developed to address both the coincidence problem and the dynamical problem. Throughout the proceeding discussion, we only deal with the late-time evolution of the Universe. Here, $H^{-1} >> \tau$ so that $e^{-\tau H}$ can, justifiably, be set to unity with the result that the effective late-time Λ is controlled by $a(t)^{-\sigma(t)}$ and the Universe evolves quasi-classically[1]. The associated vacuum energy density can, thus, be written as

$$\rho_v(t) = \frac{\Lambda(t)}{8\pi G} = \beta_0 \left(\frac{a(t_0)}{a(t)}\right)^{\sigma(t)} \rho_m^{(0)}, 2.2 \tag{2}$$

where, for convenience (in the second equality) we express the vacuum energy density in terms of the current values [10] of the matter density $\rho_m^{(0)}$ and the scale factor $a(t_0)$ and β_0 gives the current ratio of the dark energy density to the matter density. Note: in our notation the sup/sub/script 0 on a quantity denotes its current value.

B. Energy equations and particle creation

We consider the dynamical evolution of a self-gravitating cosmic medium consisting of a two-component perfect fluid. The total energy momentum tensor $T_{\mu\nu}$ for all the fields is given by

$$T_{\mu\nu} = T_{\mu\nu}^{(m)} + T_{\mu\nu}^{(vac)} = [\rho + p]\, v_\mu v_\nu + p g_{\mu\nu}, 2.3 \tag{3}$$

where $T_{\mu\nu}^{(m)}$ and $T_{\mu\nu}^{(vac)}$ are, respectively, the matter and the vacuum contributions to $T_{\mu\nu}$, $\rho = \rho_m + \rho_v$, $p = p_m + p_v$ and v_μ is the 4-velocity. Conservation of the total energy, $v_\mu T^{\mu\nu}_{\ ;\nu} = 0$, leads to

$$[\dot\rho_m + (\rho_m + p_m)\,\theta] + [\dot\rho_v + (\rho_v + p_v)\,\theta] = 0, 2.4 \tag{4}$$

where $\theta = v^\alpha_{\ ;\alpha}$ is the fluid expansion parameter and $\dot\rho$ is the derivative taken along the fluid worldline, $\dot\rho = v^\alpha \nabla_\alpha \rho$. In this treatment, while the total energy in the cosmic fluid is conserved, the individual components are, in general, not conserved. In particular, (under conditions to be discussed) the vacuum will act as a source of dissipative processes, while the matter component acts as a sink of such processes. This means that one can write

$$v_\mu T^{\mu\nu}_{(vac)\ ;\nu} = -v_\mu T^{\mu\nu}_{(m)\ ;\nu} = \Psi 2.5 \tag{5}$$

where $\Psi > 0$ is the source strength.

[1] A matter creating universe is not entirely classical.

In principle, a non-equilibrium system can involve dissipative processes, ranging from scalar fluxes like bulk viscous pressure and particle creation pressure to tensorial shear viscosity stresses and energy transport [18]. For the case under consideration, the latter are globally suppressed since the Universe, as described by the Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = -dt^2 + \frac{a\left(t\right)^2}{1-kr^2}\left[dr^2 + r^2 d\Omega^2\right], 2.6 \tag{6}$$

is isotropic and homogeneous. The contribution to the entropy source, by the remaining scalar processes, is given by [18]

$$S^{\alpha}_{;\alpha} = -\frac{\Pi\theta}{T} - \frac{\pi_c\theta}{T} - \frac{\mu\Psi}{T}, 2.7 \tag{7}$$

where π_c is the creation pressure, Π is the bulk viscous pressure, μ is the chemical potential and T is the temperature. In general, it can be shown [18, 19] that in the absence of the particle source strength Ψ, the creation pressure and the bulk viscosity become the same process and there is no particle creation.

In this article, we only pay attention to particle creating processes described by the creation pressure, π_c. Then, use of Eq. 2.5 shows Eq. 2.4 to consist of two dissipative equations

$$\left[\dot{\rho}_v + \left(\rho_v + p_v\right)\theta\right] = \pi_c\theta 2.8 \tag{8}$$

and

$$\left[\dot{\rho}_m + \left(\rho_m + p_m\right)\theta\right] = -\pi_c\theta.2.9 \tag{9}$$

We suppose that the interacting vacuum satisfies an effective equation of state of the form

$$p_\Lambda = w\rho_\Lambda, 2.10 \tag{10}$$

where $-1 \leq w \leq 0$. Note the 'unusual' upper limit $w \leq 0$ (instead of $w \leq -\frac{1}{3}$) for the interacting dark energy, signifying a matter dominated state in which the vacuum pressure hardly contributes to the dynamics of the Universe. The quantity w will have an implicit time dependence (through the fields) when particle creating processes are in force. It will be shown (see next subsection) that $w \longrightarrow -1$ in the limit $\rho_m \longrightarrow 0$. This means that in the absence of matter fields, one recovers (in a locally inertial frame), the standard Lorentz invariant vacuum. Further, we assume [19] that the newly created particles are virtually in thermal equilibrium with the existing matter fields as soon as they are created. This is reasonable in light of the approach we have adopted above which suppresses non-matter creating bulk viscosity effects. Thus, the only source of entropy is matter creation. As a result, the matter fields satisfy the usual γ-law equation of state

$$p_m = \left(\gamma - 1\right)\rho_m, 2.11 \tag{11}$$

where $\gamma = \left\{1, \frac{4}{3}\right\}$.

The density fields ρ_v and ρ_m can be determined using any two of the three equations, namely, the energy balance equation (Eq. 2.4) and the source-sink equations (Eqs. 2.8 and 2.9). To proceed, however, one requires the functional forms of both the power index function $\sigma\left(a(t)\right)$ and the effective equation of state $w\left(a\left(t\right)\right)$ for the interacting vacuum. This problem is addressed in the next section.

III. Evolution of vacuum energy density

In this section we develop the functional form of the vacuum energy density ρ_v and the pressure p_v by deriving $\sigma\left(a\right)$ and $w\left(a\right)$, and study how the evolution of these fields leads to a natural resolution of the two problems set out in Section I.

A. Creation pressure

In a vacuum dominated Universe, the total density ρ and pressure p give, $\rho+3p<0$ (which, as is known, violates the strong energy condition). The excess negative pressure accelerates the Universe. In turn, according to the cosmic equilibrium conjecture, the Universe backreacts to this non-equilibrium behavior by building inertia (matter) through the creation pressure π_c. We assume the local equilibrium hypothesis [20] that non-equilibrium quantities in the model depend locally on similar variables as the equilibrium ones. It follows then that the particle creating pressure, which must depend on the available excess pressure, will be proportional to the total energy/pressure quantity, $\rho+3p$, of the Universe. Consequently, we can write

$$\pi_c = K\left[\left(3\gamma-2\right)\rho_m - 2\rho_v\right], 3.1 \tag{12}$$

where, initially, ρ_v and ρ_m are the densities of the unperturbed background vacuum and matter fields. The vacuum energy dissipation parameter K is to be constrained by both observation and theoretical considerations. This parameter couples the vacuum energy to matter through creation and in the (idealized) limit $\rho_m \rightarrow 0$, it would probe the efficiency $\epsilon = 1-K$ of the Universe as a cosmic engine.

Our aim, here, is to relate the creation pressure π_c to the dynamical evolution of the density fields ρ_v and ρ_m. To proceed we start by building the effective equation of state for the interacting vacuum.

B. Field dilution

As the Universe expands the densities of the background matter fields dilute as $a(t)^{-3\gamma} = a(t)^{-3(1+\breve{w})}$, where $\breve{w} = \left\{0, \frac{1}{3}\right\}$. On the other hand, a dynamical interacting vacuum energy (of the functional form in Eq. 2.2) will suffer an energy deficit because it is a source of matter fields $v_\mu T^{\mu\nu}_{(vac)\ ;\nu} = \Psi \neq 0$.

The vacuum energy in a given comoving volume will appear to redshift with a dilution law $a(t)^{-3(1+w)}$. To preserve the functional behavior of the vacuum energy density, in Eq. 2.2, one must have $3\,(1+w) = \sigma\,(a\,(t))$, where the quantity $\sigma\,(a\,(t))$ is to be determined. This gives $w = -\left(1-\frac{\sigma}{3}\right)$, which leads to an effective equation of state

$$p_v = -\left(1-\frac{\sigma}{3}\right)\rho_v.3.2 \tag{13}$$

It is convenient to write the time evolution of the vacuum energy as an evolution in the scale factor $a\,(t)$. Since in the FRW models, the source of the four-velocity is the Hubble parameter, then $v^{\alpha}_{;\alpha} = \theta = 3\frac{\dot{a}}{a} = 3H$, and Eq. 2.8 transforms to

$$a\acute{\rho}_v + \sigma\rho_v = 3\pi_c, 3.3 \tag{14}$$

where $\acute{\rho}_v = \frac{d\rho_v}{da}$ and $\rho_v = \beta_0\rho_m^0\left(\frac{a_0}{a}\right)^{\sigma(a)}$. With Eqs. 3.2 and 3.3 the effective equation of state of an interacting dark energy can be written as

$$w = -1 + \left\{\frac{3K\left[\left(3\gamma-2\right)\rho_m - 2\rho_v\right] - a\acute{\rho}_v}{3\rho_v}\right\}.3.4 \tag{15}$$

C. Dissipative solutions

One expects that the asymptotic forms of the relation in Eq. 3.4 should recover the more familiar equations of state of ordinary physical fields. In particular, when the vacuum energy dominates the Universe ($\rho_v >> \rho_m$), one expects $w \to -1$. From Eq. 3.4, this limit requires that $-2K - \frac{1}{3}\left(a\frac{\acute{\rho}_v}{\rho_v}\right) = 0$, which integrates to

$$\rho_v\mid_{\rho_m\to 0} = \beta\rho_m^0\left(\frac{a_0}{a}\right)^{6K},\ (K \geq 0)\,.3.5 \tag{16}$$

On the other hand, the system evolves so that eventually there is no interaction between the vacuum and matter fields once the creation pressure π_c vanishes. This leads to the limit where effectively $w \longrightarrow 0$. Applying this limit on Eq. 3.4 gives $w = -1 - \frac{1}{3}\left(a\frac{\acute{\rho}_v}{\rho_v}\right) = 0$, so that

$$\rho_v\mid_{\pi_c\to 0} = \beta_0\rho_m^0\left(\frac{a_0}{a}\right)^3.3.6 \tag{17}$$

The above limits (Eqs. 19 and 20) establish bounds on the power index function $\sigma\,(t)$ as

$$6K < \sigma \leq 33.7 \tag{18}$$

At $\sigma = 3$, the vacuum (dark) energy has a maximum dilution rate and decouples from matter. The evolution of the dark energy to this state also implies a relative increase (and eventual domination) of the matter fields over the vacuum energy. Here, matter creation is suppressed consistent with the requirements of the cosmic equilibrium conjecture. With no more creation, the matter dilution grows towards its "normal" rate of $\sim a^{-3\gamma}$. In turn, this increases the density of the vacuum energy relative to that of the matter fields, eventually setting the vacuum energy into dominance. The Universe then begins to accelerate and, in the process, builds the creation pressure π_c, as a backreaction to the acceleration. According to Eqs. 3.5 and 3.7 this creation rate will grow, maximizing at $\sigma_{\min} = 6K$. As the matter creating vacuum dumps in more and more matter, the matter fields are evolving with a decreasing dilution rate. The relative increase in the density of matter fields, makes it less and less favorable for the vacuum to create more matter. As a result, the creation rate is decreasing towards a minimum. Eventually, the system ends up back where it started with $\rho_m \sim a^{3\gamma}$ and $\pi_c \sim 0$ and a new cycle begins.

It follows that, in general, the power index function $\sigma(t)$ will be oscillatory within the bounds $6K < \sigma(t) \leq 3$ as established in Eq. 3.7. Oscillations in $\sigma(t)$ naturally imply oscillations in the density function, $\rho_v \sim a^{-\sigma(t)}$. Here, it should be pointed out that while the dark energy density function (we seek) is oscillatory, it must still be single-valued in $a(t)$ in order to be consistent with the requirement that (globally) matter/entropy creation from the vacuum is an irreversible process. The vacuum energy decays into matter but not vise versa. Thus such oscillations will be imprinted on a decaying energy background. As is shown below, this is an inherent feature of the model.

The oscillations in ρ_v signify matter creation from the vacuum. This implies that such periodic matter creation will necessarily induce sympathetic oscillations in the matter density fields ρ_m. It is in this sense that the two fields, in time, track each other. To support these assertions we start by studying how the creation pressure will drive the oscillations. For convenience, we rewrite the vacuum energy density as $\rho_v = \bar{\beta}\left(\frac{\alpha}{a}\right)^{\sigma(t)}$, where $\alpha = a_0$ and $\bar{\beta} = \beta_0 \rho_m^0$. On taking derivatives with respect to $a(t)$, we have that $\acute{\rho}_v(a) = \left[\acute{\sigma}\ln\left(\frac{\alpha}{a}\right) - \frac{\sigma}{a}\right]\rho_v$. Substituting for $\acute{\rho}_v$ in Eq. 3.3 gives

$$\acute{\sigma} a \ln\left(\frac{\alpha}{a}\right) = \frac{3\pi_c}{\rho_v}.3.8 \tag{19}$$

Now, consider $\sigma(\psi)$, where $\psi(a)$ is some function of $a(t)$. Then $\acute{\sigma}\frac{da}{d\psi} = \frac{d\sigma(\psi)}{d\psi}$. Comparing this with Eq. 3.8 we find, on setting $\frac{d\psi}{da} = \left[a\ln\left(\frac{\alpha}{a}\right)\right]^{-1}$, that

$$d\sigma = \left(\frac{3\pi_c}{\rho_v}\right)d\psi.3.9 \tag{20}$$

Thus, $\frac{3\pi_c}{\rho_v}d\psi$ is a perfect differential of the power index function σ.

One expects solutions to Eq. 3.9 with certain specific characteristics. First, from the discussion above, such solutions should be oscillatory in ψ and also bounded by Eq. 3.7. Secondly, all the $\psi(a(t))$ dependence in the solution σ should be purely sinusoidal, so as to avoid unphysical solutions of the form $\rho \sim a^{f(a)}$, where $f(a)$ is non-oscillatory. Finally, as is pointed out in the cosmic equilibrium conjecture, matter creation (in this model) is an opposite reaction to the vacuum-induced *positive* acceleration of the Universe. Thus, the sinusoidal part of $\frac{d\sigma}{d\psi}$ should be negative definite, i.e. of the form $\sim \left(-\sin^2\psi\right)$, at the least.

The simplest solution that satisfies these conditions has the form $\sigma = \sin 2\psi + A$, where A is a positive constant. Only then are the requirements on $\frac{d\sigma}{d\psi}$ satisfied, since now $\left\{\frac{d\sigma}{d\psi} = \frac{3\pi_c}{\rho_v} = 3K\left[\frac{(3\gamma-2)\rho_m - 2\rho_v}{\rho_v}\right]\right\} = -4\sin^2\psi + 2$. On rearranging out the terms in the last equality one finds that

$$\sin^2\psi = \left[\frac{2\left(1+3K\right)\rho_v - 3K\left(3\gamma-2\right)\rho_m}{4\rho_v}\right].3.10 \tag{21}$$

This function is minimum $\left(\sin^2\psi = 0\right)$ at

$$_{\min}\rho_v = \left(\frac{3K}{6K+2}\right)\left(3\gamma-2\right)\rho_m, 3.11 \tag{22}$$

and maximum $\left(\sin^2\psi = 1\right)$ at

$$_{\max}\rho_v = \left(\frac{3K}{6K-2}\right)\left(3\gamma-2\right)\rho_m.3.12 \tag{23}$$

We soon return to these limits in the next section. To formally complete the solution ($\sigma = \sin 2\psi + A$), one notes on applying the limits from Eq. 3.7 that when $\sin 2\psi = 1$, then $1 + A = 3$. Thus the full solution becomes

$$\sigma\left(\psi\right) = 2 + \sin 2\psi.3.13 \tag{24}$$

Eq. 3.13 implies that the interacting vacuum energy density evolves with the scale factor as

$$\rho_v = \bar{\beta}\left(\frac{\alpha}{a}\right)^{(2+\sin 2\psi)}.3.14 \tag{25}$$

The functional form of ρ_v in Eq. 3.14 exhibits the following characteristics. During the evolution of the Universe, the vacuum field has the highest dilution rate (and is least interacting) at points characterized by $\psi_n = n\pi,\ \{n = 0, 1, ...\}$. It is at these points that the interacting vacuum energy decouples from the matter fields so that $w \longrightarrow 0$. On the other hand, the field has its least dilution rate (and is most interacting) form at epochs characterized by $\psi_n = \left(\frac{1}{2} + n\right)\pi,\ \{n = 0, 1, 2, ...\}$. In general, Eq. 3.14 shows that in its dynamical form, the vacuum energy density fluctuates about a^{-2}, crossing this value when $\psi_n = \left(\frac{1}{4} + n\right)\pi,\ \{n = 0, 1, ...\}$.

Note that since $< \sin 2\psi > \ = 0$ and $\psi = \psi\left(a\left(t\right)\right)$, then Eq. 3.13 gives the time average of the power index function as $< \sigma >= 2$. Eqs. 3.13 and 3.14 imply that the mean vacuum energy density $< \rho_v >$ evolves as a^{-2}. Thus, in an expanding Universe, an interacting vacuum is inherently a decaying system. Such a feature guarantees that (even with an oscillatory power index) the density function in Eq. 3.14 is single-valued. In turn, this ensures the dissipations leading to matter creation are a one way process and so is the entropy growth.

Finally, note that one now has the tool needed to address the question "why is Λ small now" which was first brought up in Section II. Thus, $<\sigma>= 2 \Longrightarrow \left[\Lambda_{now} \sim a_0^{-2}\right] \approx 10^{-120}\Lambda_{pl}$. Such a result is consistent with observations.

That $<\rho_v>$ evolves as a^{-2} deserves some further comment. In the past, several phenomenological models have been developed (see [21] and citations) in which the vacuum energy density ρ_v evolves with the scale factor as a^{-m}, where the index m has a fixed value. In such models the vacuum energy density is not explicitly interacting, as is indicated by the constant nature of the index m. One such set of models that has gained considerable popularity (see citations in [21]) evolves ρ_v as an inverse square power law ($m = 2$), in the scale factor. It is worthy noting that the approach presented, recovers this inverse square power law as the mean value of an interacting vacuum energy density, $<\rho_v>$,

The last feature of the solutions (Eqs. 3.13 and 3.14) to consider relates to the coupling parameter, K. Eq. 3.10, indicates that in the limit $\rho_m \to 0$ (i.e. in the absence of matter fields), the dissipation parameter has an upper bound so that

$$K\mid_{\rho_m\to 0} \quad \not> \frac{1}{3} 3.15 \tag{26}$$

On the other hand, in the presence of matter, Eq. 3.12 clearly indicates that creation will take place only if $K > \frac{1}{3}$. Note that in Eq. 3.12, ${}_{\max}\rho_v$ is always finite since, evidently, ρ_m must vanish at $K = \frac{1}{3}$. Further, observations (e.g. [2]) suggest that currently $\rho_v \approx 2\rho_m$. Applying this to Eq. 3.12 provides an upper bound for K so that at most, $K \lesssim \frac{4}{9}$. Combining these two bounds, from both observational evidence and theoretical considerations, one finds for matter (massive particle) creation, the dissipation parameter is constrained to

$$\frac{1}{3} < K \lesssim \frac{4}{9} . 3.16 \tag{27}$$

This constraint is not as effectively narrow as it looks since, as one can infer from Eq. 3.12, ${}_{\max}\rho_v$ is quite sensitive to small variations in K. The information in both Eqs. 3.15 and 3.15, when taken together, suggests that for the dissipation parameter space $0 < K \leq \frac{1}{3}$ particle creation may still be possible, provided (to be consistent with Eq. 3.12) such particles are massless. Consequently, as one would expect, the dissipation parameter K is always associated with entropy production.

One can imagine a time in the very early history of the Universe when all the vacuum energy therein was very large and purely potential (PE). Such conditions would imply that, at the time, $K = 0$. Then $\rho_v \sim a^{-(6K)=0}$ and the vacuum energy density is constant in time. Only then, albeit temporarily, would the Universe as an engine operate at 100% efficiency. Clearly this is the energy in a Cosmological Constant and it would inflate the Universe. In turn, because inflation would tend to strongly and suddenly move the Universe away from equilibrium conditions, the cosmic equilibrium conjecture demands that the associated backreation also be strong and sudden. Accordingly, there are two consequences to this. First, it is in immediate reaction to this scenario that most of the present inertia (matter) and entropy is created, (starting with the massless particles as the dissipation parameter grows from $K = 0$ to $K = \frac{1}{3}$, to the massive particle production ($K > \frac{1}{3}$)). Note that this suggests a temporal ordering, in particle production. In turn, it is precisely the growth of such inertia (i.e. $K \to \frac{1}{3}$) that would lead the Universe to a graceful exit from inflation. The inflationary scenario would probably last as long as it would take for the dissipation parameter to grow to $K \to \frac{1}{3}$. Thus, in this model, inflation and its immediate self destruction are natural initial conditions to the current evolution of the Universe.

IV. Consequences of vacuum decay

A. Evolution of the matter fields density, $\rho_m(a)$

The preceding analysis for the evolution of ρ_v has been based on the source equation (Eq. 2.8). In order to discuss the evolution of the matter fields density $\rho_m(a)$ in the presence of a creation pressure π_c, the above results can be introduced either in the energy balance equation (Eq. 2.4) or the sink equation (Eq. 2.9). Choosing the latter and rewriting Eq. 2.4 as a function of $a(t)$ one finds

$$a\acute{\rho}_m + 3\gamma\rho_m + 3\pi_c = 0, 4.1 \tag{28}$$

where we have used Eq. 2.11 and $\theta = 3H$. From Eqs. 3.10,

$$3\pi_c = 2\left(1 - 2\sin^2\psi\right)\rho_v, 4.2 \tag{29}$$

where ρ_v is given by Eq. 3.14. Introducing these results in Eq. 4.1 gives

$$a\acute{\rho}_m + 3\gamma\rho_m + 2\left(1 - 2\sin^2\psi\right)\left[\bar{\beta}\left(\frac{\alpha}{a}\right)^{(2+\sin 2\psi)}\right] = 0, 4.3 \tag{30}$$

where, as previously established, $\frac{d\psi}{da} = \left[a\ln\left(\frac{\alpha}{a}\right)\right]^{-1}$. Eq. 4.3 governs the evolution of the matter fields density $\rho_m(a)$ in the model.

In this article the main aim has been to build a framework for discussing both the cosmological constant problem and the dynamical problem. In the previous section we have touched on the question of "why Λ is small now". It still remains to address both the coincidence problem and the dynamical problem. As it turns out, the results of the preceding section are sufficient for such a discussion. Consequently, we defer a discussion of the solutions to Eq. 4.3 to concentrate on the two remaining problems.

B. The coincidence problem and the dynamical problem

Using the results of Eqs. 3.10 to 3.12 one finds that the vacuum dilution rates relate directly to the vacuum to matter ratios in the Universe. In particular, at $\sin^2\psi = 0$, when the vacuum is at its most dilution rate, Eq. 3.11 constrains the minimum density value of the vacuum to ${}_{\min}\rho_v = \left(\frac{3K}{6K+2}\right)(3\gamma-2)\rho_m$. On the other extreme at $\sin^2\psi = 1$, when the vacuum is at its least dilution rate, Eq. 3.12 constrains the maximum density value of the vacuum to ${}_{\max}\rho_v = \left(\frac{3K}{6K-2}\right)(3\gamma-2)\rho_m$. These results imply that as the Universe evolves, the vacuum and matter fields are coupled through the coupling parameter K, which (see Eq. 3.16) is constrained. The vacuum then tracks the matter fields naturally within the bounds given by Eqs. 3.11 and 3.12 as

$$\left(\frac{3K}{6K+2}\right)(3\gamma-2)\rho_m \leq \rho_v \leq \left(\frac{3K}{6K-2}\right)(3\gamma-2)\rho_m .4.4 \tag{31}$$

In particular, during the radiation era $\gamma = \frac{4}{3}$, the vacuum oscillates between the values $\left(\frac{3K}{3K+1}\right)\rho_m \leq \rho_v \leq \left(\frac{3K}{3K-1}\right)\rho_m$. On the other hand, during the cold matter era $\gamma = 1$, the vacuum oscillates between the values $\left(\frac{3K}{6K+2}\right)\rho_m \leq \rho_v \leq \left(\frac{3K}{6K-2}\right)\rho_m$. Clearly, provided $K > \frac{1}{3}$, the vacuum and matter field densities, ρ_v and ρ_m will track each other naturally, thus, providing a resolution to the coincidence problem. Moreover, as long as the Universe expands, these fields track each other with decaying amplitudes.

Finally, since the dynamics of the Universe is determined by the behavior of the fields therein, the foregoing results can be used to predict the future evolution of the Universe. In this sense, the results also address the "dynamical problem".

C. Observational tests

The model predicts that the Universe undergoes periods of variable acceleration. In the end the maximum relative amplitudes of the density fields can only be determined with knowledge of the exact value of the free parameter K. Here, we have only been able to constrain K to $\frac{1}{3} < K \lesssim \frac{4}{9}$. As was mentioned before ${}_{\max}\rho_v$ is quite sensitive to small variations in K. In the unlikely event that the Universe is currently at a local maximum of vacuum domination, so that ${}_{\max}\rho_v : \rho_m \approx 2$, then one finds $K \approx \frac{4}{9}$. The most probable scenario, however, is that the current vacuum energy density ρ_v^0 is not at a local maximum ${}_{\max}\rho_v$ of relative vacuum domination. This implies that the vacuum energy could take on larger dominant role and $K < \frac{4}{9}$. In this case the local dynamical evolution of the Universe has two possibilities (Eq. 3.10): the Universe is either moving away or towards a local ${}_{\max}\rho_v$. Such an ambiguity can only be resolved by observation. Thus, one of the observational evidences to look for is variability the acceleration of the Universe as a function of redshift $z = \left(\frac{a_0}{a} - 1\right)$. An increase in acceleration with z, for example, would indicate the Universe is moving away from a local maximum of ${}_{\max}\rho_v : \rho_m$. All in all, tracking such a maximum would also help constrain the Universe's general dynamics by further constraining K. One hopes that future space-based observational projects like SNAP [22] will shed some light on these issues.

V. Conclusion

In this paper a framework is proposed for addressing both the cosmological constant problem and the associated dynamical problem. The underlying premise is that the background dark energy is both dynamical and interacting. Such a feature gives rise to a Universe that behaves as a cosmic thermodynamic engine, in which the vacuum energy is the input fuel. The vacuum energy does work by expanding the Universe. However, like any engine, it is impossible for the Universe to use all the vacuum energy to do work without some of it being dissipated. The rationale for this dissipation and for the implied matter creation is embodied in a "cosmic equilibrium" conjecture we make with regard to the need for the Universe to seek for equilibrium conditions. The agent for the matter creation is a creation pressure π_c that arises as a backreaction to the cosmic acceleration. In creating matter, this pressure also creates entropy so that the Universe as a cosmic engine satisfies both the first and second laws of thermodynamics.

As pointed out in Sec. II the "why is Λ small now?" part of the cosmological constant problem is addressed by noting that in this model $<\sigma>=2$, so that $<\rho_v>$ evolves as $\sim a^{-2}$. Such inverse square behavior was established in Sec. III. From the inflation era to date, $a(t)$ has evolved by about 10^{60}. This implies that $\Lambda_{now} \approx 10^{-60<\sigma>}\Lambda_{pl} = 10^{-120}\Lambda_{pl}$, a result which is consistent with observations.

It was discussed, in some detail, how the vacuum energy density ρ_v couples to the matter fields ρ_m through matter creation pressure π_c. This coupling is facilitated by a parameter K which (for the post-inflationary period) we have constrained to $\frac{1}{3} < K \lesssim \frac{4}{9}$. The gives rise to a vacuum energy density which oscillates with a decaying amplitude as it creates matter. In turn, the matter fields oscillate in sympathy. The result is that these two coupled fields track each other naturally. We have put bounds (Eq. 4.4) on the relative evolution of the magnitudes of these fields up to the constrained free parameter K. It is in this sense that the model addresses the coincidence problem. The bounds put on the relative magnitudes of these fields (Eq.4.4) also imply that the future evolution of these fields is predictable. Since it is these fields that drive the Universe, in the first place, the result is that the future dynamics of the Universe becomes equally predictable. In this way the model addresses the dynamical problem.

It is pointed out that the coupling parameter K must have, at one time, had to grow from zero to its minimum operative value $\frac{1}{3}$. This growth correlates with the vacuum energy changing from a purely potential form at $K = 0$ to a partially dissipated form $K > 0$. During this growth (as long as $K < \frac{1}{3}$), there would be little or no matter created and the Universe, essentially, behaves as a near-perfect engine with $\sim 100\%$ efficiency. The vacuum energy is then, mostly, in the form of a cosmological constant. During this initial period (i.e. in the neighborhood of $K = 0$) the Universe must inflate. Consequently, our treatment requires inflation as a natural initial condition, both for creation and for the current classical dynamics of the Universe. Moreover, the growth of the dissipation parameter (and hence that of the creation pressure) as a backreaction to the inflationary acceleration, creates a natural graceful exit out of inflation through increase of the Universe's internal energy. Thus, in this model, the cosmic equilibrium conjecture demands that inflation (through dissipative processes) oversee its own destruction, to end almost immediately. It is in this sense that the approach predicts both inflation and a graceful exit, while at the same time justifying matter creation. Further, using the same mechanism, the Universe enters and eventually exits from any subsequent accelerations, including the current one.

The model proposed here makes testable predictions that the dynamics of the Universe is variable with a quasi-periodic character. A detailed discussion of the observational implications will be explored in a future work.

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